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MIM Capacitor Modeling: A Planar Approach

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***Abstract*—On the basis of a planar approach, an equivalent circuit model of MMIC overlay capacitors with feeding lines in generic position has been derived. Closed expressions for the equivalent circuit elements are presented. The validity of the proposed circuit has been successfully tested with experimental data. Its simplicity and flexibility makes it attractive for implementation in CAD packages.**

I. Introduction

Metal–Insulator–Metal (MIM) capacitors are key elements in many microwave and millimeter-wave monolithic integrated circuits. DC-blocks, matching sections, and biasing circuitry widely utilize this component. An accurate model of the structure is therefore crucial for any MMIC design. Many monolithic foundries have developed their own proprietary models by means of parameter extraction methods from experimental data. The so-obtained models are too device-specific and their validity is restricted to the frequency range of the fitting measurements. Other approaches, closer to the physical structure of the MIM capacitor, have been previously presented, resulting in a distributed [1] or lumped [2] equivalent model. Both approaches are zero- or mono-dimensional and therefore do not take into account the intrinsic planar behavior of the structure, demonstrated in [3] and [4]. Such a planar behavior must be accounted for when, due to layout constraints, the designer is forced to feed the capacitor with off-centered ports or even positioned on perpendicular sides of the structure.

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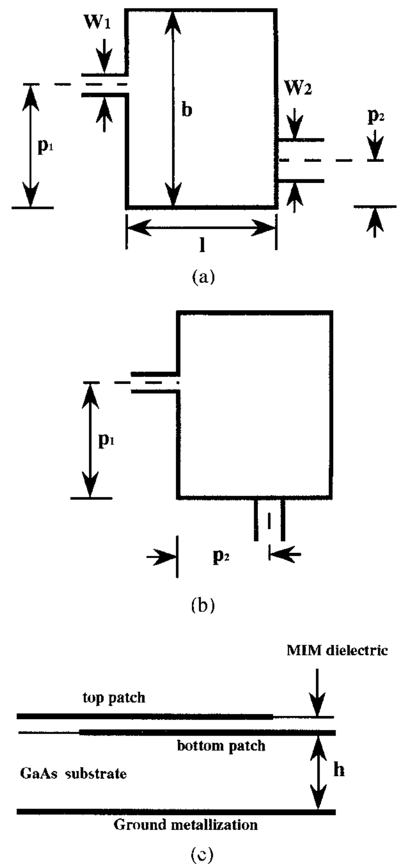

Fig. 1. The MIM capacitor: (a) top view for feeding lines on opposite sides; (b) perpendicular sides; and (c) section of the structure.

Moreover, design considerations suggest the development of a circuit model of the MIM capacitor that can be easily implemented in commercial CAD packages. To fulfill these requirements, in this paper a novel circuit model, based on a planar approach, for two-port overlay capacitors is presented. The model accounts for the effects of generic port positioning: the ports can be placed on opposite sides of the structure and off-centered or even on perpendicular sides. Simple and closed-form expressions for the model elements are provided. The proposed model has been successfully tested comparing the simulated results to experimental data in a wide frequency band.

II. The Equivalent Circuit

The typical structure of an MIM capacitor is shown in Fig. 1. The prevailing effect at very low frequencies is by far the capacitive one: it can be easily represented by a series-connected parallel-plate capacitor. As frequency increases, planar effects due to the bottom patch, which are not taken into account by this simple model, become evident, altering the ideal capacitive behavior. This leads us to model the whole structure as a cascade connection of the main series capacitor (C_{MIM}, whose value can be computed as if it were a parallel-plate capacitor) and of the bottom patch, added to account for the above-mentioned planar effects.

As already well established (see [5]), the external behavior of such a planar structure can be modeled in terms of a Z-matrix description. The entries Z_{ijR} are obtained enclosing the patch by lateral magnetic walls and expanding the EM field inside the resulting resonator into

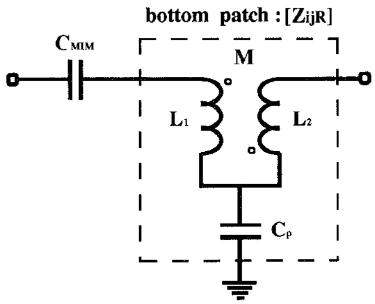

Fig. 2. Equivalent-circuit model of the MIM capacitor.

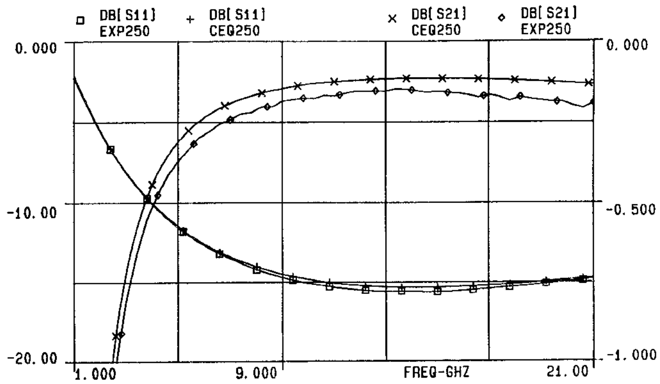

Fig. 3. dB$[S_{11}]$ and dB$[S_{21}]$ versus frequency for a $250 \times 250\,\mu m^2$ MIM capacitor: simulations and experiments.

a series of resonant modes. With this approach, the entire structure can be represented as follows:

$$[Z] = \begin{bmatrix} \frac{1}{j\omega C_{MIM}} + & Z_{11R} & Z_{12R} \\ & Z_{21R} & Z_{22R} \end{bmatrix}$$

where the generic entry $Z_{ijR}(i, j = 1, 2)$ can be written [7] as

$$Z_{ijR} = \frac{h}{j\omega\epsilon_0\epsilon_{00}}\frac{1}{bl} + j\omega\mu_0 h \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\delta_m\delta_n\theta_{mnij}}{bl(k_{mn}^2 - \omega^2\mu_0\epsilon_0\epsilon_{mn})}$$

where m, n cannot be simultaneously equal to zero, and h is the substrate height; b, l, w_i are the effective dimensions corresponding to the physical ones and computed after [6]; ϵ_{mn} is the effective permittivity of the (m, n)th mode [6];

$$k_{mn}^2 = \pi^2\left[\left(\frac{m}{l}\right)^2 + \left(\frac{n}{b}\right)^2\right] \qquad m, n = 0, 1, 2, \cdots$$

$$\delta_m = \begin{cases} 1 & \text{if } m = 0 \\ 2 & \text{if } m \neq 0 \end{cases}$$

and, in the case of feeding lines on opposite sides of the structure (Fig. 1(a))

$$\theta_{mnij} = (-1)^{m(i+j)} \cdot f_{in} \cdot f_{jn}$$

while, for feeding lines on perpendicular sides (Fig. 1(b))

$$\theta_{mnij} = \begin{cases} f_{1n}^2 & \text{if } i = j = 1 \\ f_{1n} \cdot g_{2m} & \text{if } i = 1, j = 2 \text{ or } i = 2, j = 1 \\ g_{2m}^2 & \text{if } i = j = 2 \end{cases}$$

with

$$f_{in} = \begin{cases} \cos\left(\frac{n\pi p_i}{b}\right) \cdot \frac{\sin\left(\frac{n\pi w_i}{2b}\right)}{\frac{n\pi w_i}{2b}} & \text{if } n \neq 0 \\ 1 & \text{if } n = 0 \end{cases}$$

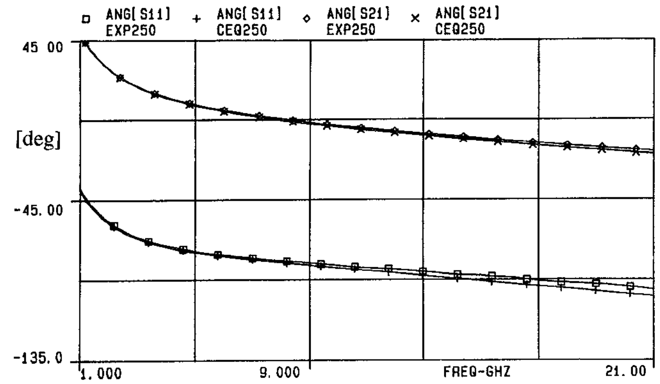

Fig. 4. ang$[S_{11}]$ and ang$[S_{21}]$ versus frequency for a $250 \times 250\,\mu m^2$ MIM capacitor: simulations and experiments.

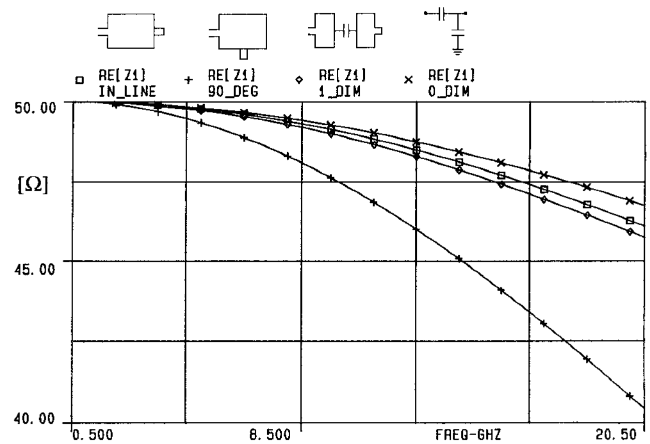

Fig. 5. Real part of the input impedance of a series-connected MIM capacitor terminated with a 50-Ω load: comparison among the proposed model with centered ports on opposite sides, with centered ports on perpendicular sides, and one- and zero-dimensional models.

$$g_{im} = \begin{cases} \cos\left(\frac{m\pi p_i}{l}\right) \cdot \frac{\sin\left(\frac{m\pi w_i}{2l}\right)}{\frac{m\pi w_i}{2l}} & \text{if } m \neq 0 \\ 1 & \text{if } m = 0 \end{cases} \qquad i = 1, 2.$$

In the present case, this general approach can be significantly simplified: the geometric dimensions of commonly used MIM capacitors result in resonating frequencies much higher than the operating frequency range. It means that the following relation holds:

$$k_{mn}^2 \gg \omega^2\mu_0\epsilon_0\epsilon_{mn}.$$

The generic matrix entry Z_{ijR} can therefore be rewritten as

$$Z_{ijR} = \frac{h}{j\omega\epsilon_0\epsilon_{00}}\frac{1}{bl} + j\omega\mu_0 h \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\delta_m\delta_n\theta_{mnij}}{blk_{mn}^2}. \tag{1}$$

A simple inspection of the above formula gives a major insight into the electrical behavior of the planar portion: both the terms *depend on the frequency only explicitly,* thus allowing a direct interpretation in terms of lumped elements. More precisely, the expressions in (1) can be easily recognized to be the Z parameters of a purely reactive T network. The resulting equivalent circuit is shown in Fig. 2, where its elements have the following expressions:

$$C_P = bl\epsilon_0\epsilon_{00}/h$$

$$L_i = \frac{Z_{iiR} - \frac{1}{j\omega C_P}}{j\omega} = \mu_0 h \sum_{m}^{M}\sum_{n}^{N} \frac{\delta_m\delta_n\theta_{mnii}}{blk_{mn}^2} \qquad i = 1, 2$$

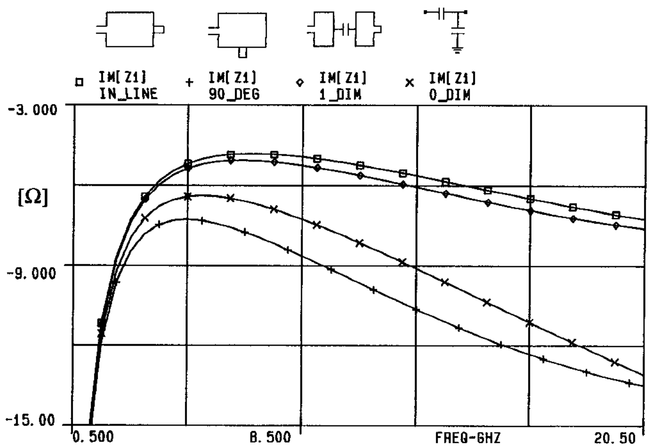

Fig. 6. Imaginary part of the input impedance of a series-connected MIM capacitor terminated with a 50-Ω load: comparison among the proposed model with centered ports on opposite sides, with centered ports on perpendicular sides, and one- and zero-dimensional models.

$$M_P = \frac{Z_{12R} - \frac{1}{j\omega C_P}}{j\omega} = \mu_0 h \sum_m^M \sum_n^N \frac{\delta_m \delta_n \theta_{mn12}}{blk_{mn}^2}.$$

Moreover, these elements are directly related to the physical behavior of the structure: C_p takes into account the electrostatic effects of the bottom patch, and L_1 and L_2 account for the interactions between the quasi-TEM mode on the feeding lines and the higher-order modes excited inside the patch; the inductive terms are therefore related to the feeding lines discontinuities and are responsible for the reactive energy storage. On the other hand, the coupling coefficient M_p represents the interaction between the two discontinuities supported by the higher-order modes. It is worth noting the following:

- The summation over m and n can be truncated to finite values M, N; it is wise to choose the ratio M/N as close as possible to the ratio l/b in order to have the same highest spatial frequencies in both directions. Moreover, the resonating frequencies are usually quite far from the operating ones, and the contribution of very high-order modes is negligible, so allowing to truncate the summations to a few modes (usually MIN$(M, N) \leq 10$).
- If the operating frequency approaches the first resonating one, the corresponding mode can be taken into account adding a parallel LC cell [7] to the residual inductance.
- All the equivalent circuit elements are frequency-independent, thus obtaining a truly lumped model.

III. Results

In order to check the validity of the proposed equivalent circuit, several test structures on a 200-μm-thick GaAs substrate ($\epsilon_r = 12.9$) were realized with different sizes and filling dielectrics (namely, polymide and silicon nitride). The feeding lines are centered and with $W_1 = W_2 = 30\mu$ m. For the sake of brevity, only the results for a $250 \times 250\mu\text{m}^2$ (i.e., 1.3 pF) polymide capacitor are presented in Figs. 3 and 4 in the form of scattering parameters. The good agreement between simulated and measured results demonstrates the validity of the proposed equivalent circuit.

The suggested planar approach can be used to account for "shape effects" that cannot be investigated using different modeling strategies. In fact, due to layout constraints, the designer is often forced to use nonsquare geometries for the MIM capacitors, and must simulate the resulting structure as if it were a square one, which is normally considered in foundry models; in this case, the planar behavior of the capacitor may deviate substantially from the predicted one.

In order to show the capabilities of the proposed planar approach, the developed model has been compared to zero- and one-dimensional models for a 10 pF capacitor (with $W_1 = W_2 = 20\mu$ m, $b = 120\,\mu$m, $l = 360\,\mu$m, $h = 120\,\mu$m). The zero-dimensional model is a simple L-section containing the main capacitor (series-connected capacitor C_{MIM}) and the geometrical capacitance of the bottom plate to ground (shunt-connected capacitor). The one-dimensional model is a simple transmission line-capacitor-transmission line model, in which the transmission line length is half the capacitor longitudinal dimension after adding the step discontinuities at both ends of the capacitor [8]. The proposed planar model has been applied to the two different cases of centered feeding lines on opposite sides and centered lines on perpendicular sides (Fig. 1(a) and (b)). The results obtained from the four resulting equivalent circuits, corresponding to the input impedance when the second port is terminated in a 50-Ω load, are plotted in Figs. 5 and 6. This is actually the case of the terminating impedance of a matched stage, where the MIM capacitor acts as a DC-block. It is worth noting the good agreement between the results of the planar model for the opposite side ports case and the one-dimensional model; on the other hand, the results for the case of ports on perpendicular sides deviate substantially from the previous ones, evidencing the influence of a generic port positioning on the behavior of the structure and stressing the need for a truly planar model.

IV. Conclusion

An equivalent circuit model, based on a planar approach, for the Metal–Insulator–Metal capacitor has been presented. The model accounts for generic port positioning, and its elements can be obtained in closed form via simple formulas and are frequency-independent. The excellent agreement with experimental results, obtained over a broad frequency range, demonstrates the effectiveness of the proposed approach.

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